UNIVERSITY of VIRGINIA | THE DEMOGRAPHICS RESEARCH GROUP

State and National Projections Methodology

This document describes in detail the steps in developing projections for the nation, the 50 states, and the District of Columbia for 2020, 2030, and 2040. The projections include:

- Total population
- By five-year age group
- By gender

We use primarily the Hamilton-Perry approach, which is a reduced form of the cohort-component method (Hamilton and Perry, 1962). Detailed national characteristics are equal to the sum of the detailed projections for each of the 50 states and the District of Columbia. State population projections are controlled to the national total; age is controlled to the state total population; and sex is controlled to each age-group.

TOTAL POPULATION PROJECTIONS

DATA:

- U.S. population for
  - 1990 (1 April count) – by 18 age cohorts
  - 2000 (1 April count) – by 18 age cohorts
  - 2010 (1 April count) – by 18 age cohorts and total
  - 2015(1 July estimate by Census Bureau) – total
- 50 states and D.C. population for
  - 2010 (1 April count) – total
  - 2015 (1 July estimate by Census Bureau) – total

APPROACH FOR 2020 TOTAL POPULATION:

- Calculating population growth rate for each of the 50 states and D.C., using data from 2010 and 2015.

$$StatePopulation_{2015} = StatePopulation_{2010} * e^{r*5.25}$$

which implies the rate is

$$r = \frac{1}{5.25} * \ln(\frac{StatePopulation_{2015}}{StatePopulation_{2010}})$$

- Setting the final state level population projection for 2020, after calculating the projected state population for 2020 from exponential growth, using data from 2015.[1]

Using 2015 as the launch year,

$$StatePopulationExp_{2020} = StatePopulation_{2015} * e^{r*4.75}$$

This is the final state level population projection for 2020,

$$\boldsymbol{StatePopulation_{2020}} = StatePopulationExp_{2020}$$

- Calculating the projected U.S. population for 2020, from summing state populations.[2,3]

This is the final national level population projection for 2020.

$$\boldsymbol{USPopulation_{2020}} = \sum_{States} StatePopulation_{2020}$$

APPROACH FOR 2030 AND 2040 TOTAL POPULATION:

- Generating CPRs (child population ratio) and CCRs (cohort change ratio) for each age cohort of the U.S. population, using data from 1990, 2000, and 2010.

For cohorts 0-4 and 5-9, CPRs capture the birth rates in the prior decades. We divide the child population by the appropriate population of child-bearing age to generate the CPRs.

$$CPR_{0-4}^{1990-2000} = \frac{USPopulation_{0-4}^{2000}}{USPopulation_{15-44}^{2000}} \quad \& \quad CPR_{0-4}^{2000-2010} = \frac{USPopulation_{0-4}^{2010}}{USPopulation_{15-44}^{2010}}$$

$$CPR_{5-9}^{1990-2000} = \frac{USPopulation_{5-9}^{2000}}{USPopulation_{20-49}^{2000}} \quad \& \quad CPR_{5-9}^{2000-2010} = \frac{USPopulation_{5-9}^{2010}}{USPopulation_{20-49}^{2010}}$$

For cohorts 10-14, …. 60-64, 85+, CCRs measure the combined effects of deaths and migration. We use the ratio of population in an age-group (*a*) in one decade, to the population in age-group (*a-10*) in the previous decade, to calculate CCRs.

$$CCR_{Age\ Cohort}^{1990-2000} = \frac{USPopulation_{Age\ Cohort}^{2000}}{USPopulation_{Age\ Cohort-10}^{1990}} \quad \& \quad CCR_{Age\ Cohort}^{2000-2010} = \frac{USPopulation_{Age\ Cohort}^{2010}}{USPopulation_{Age\ Cohort-10}^{2000}}$$

To smooth out fluctuations, we use averaged CCR and CPR values over 1990-2000 and 2000-2010.

$$\overline{CPR}_{Age\ Cohort} = \frac{CPR_{Age\ Cohort}^{1990-2000} + CPR_{Age\ Cohort}^{2000-2010}}{2} \quad \& \quad \overline{CCR}_{Age\ Cohort} = \frac{CCR_{Age\ Cohort}^{1990-2000} + CCR_{Age\ Cohort}^{2000-2010}}{2}$$

For cohorts 65-69, …. 80-84, 85+, CCRs are adjusted to account for the effect of an aging population

$$\overline{CCR}_{Age\ Cohort} = \frac{\frac{USPopulation_{Age\ Cohort}^{2010}}{USPopulation_{Age\ Cohort-10}^{2000}}}{\frac{USPopulation_{Age\ Cohort}^{2000}}{USPopulation_{Age\ Cohort-10}^{1990}}} * \frac{USPopulation_{Age\ Cohort}^{2010}}{USPopulation_{Age\ Cohort-10}^{2000}}$$

- Calculating the projected U.S. population for 2020 from Hamilton-Perry age forwarding, using data from 2010 and calculated average CPRs and CCRs.[4]

$$USPopulationHP^{2020}_{Age\ Cohort} = \frac{USPopulation^{2010}_{Age\ Cohort}}{USPopulation^{2000}_{Age\ Cohort-10}} * USPopulation^{2010}_{Age\ Cohort-10}$$

$$= CCR^{2000-2010}_{Age\ Cohort} * USPopulation^{2010}_{Age\ Cohort-10}$$

The CCR (or CPR) is replaced by the averaged out values over 1990-2000 and 2000-2010,

$$USPopulationHP^{2020}_{Age\ Cohort} \cong \overline{CCR}_{Age\ Cohort} * USPopulation^{2010}_{Age\ Cohort-10}$$

Now the total U.S. population for 2020 from Hamilton-Perry method can be calculated by summing across all age-cohorts,

$$USPopulationHP_{2020} = \sum_{Age\ Cohorts} USPopulationHP^{2020}_{Age\ Cohort}$$

- Calculating the projected U.S. population for 2020 for each age-interval, by redistributing the U.S. population for 2020 as per the age-distribution of the U.S. population for 2020 from Hamilton-Perry method.

$$USPopulation^{2020}_{Age\ Cohort} = \frac{USPopulationHP^{2020}_{Age\ Cohort}}{USPopulationHP_{2020}} * USPopulation_{2020}$$

- Setting the final national level population projection for 2030, after calculating the projected U.S. population for 2030 from Hamilton-Perry age forwarding.[5,6]

$$USPopulationHP^{2030}_{Age\ Cohort} \cong \overline{CCR}_{Age\ Cohort} * USPopulation^{2020}_{Age\ Cohort-10}$$

Now the total U.S. population for 2030 from Hamilton-Perry method can be calculated by summing across all age-cohorts,

$$USPopulationHP_{2030} = \sum_{Age\ Cohorts} USPopulationHP^{2030}_{Age\ Cohort}$$

This is the final national level population projection for 2030,

$$\boldsymbol{USPopulation_{2030}} = USPopulationHP_{2030}$$

- Setting the projected U.S. population for 2030 for each age-interval, as per the age-distribution of the U.S. population for 2030 from Hamilton-Perry method.

$$USPopulation^{2030}_{Age\ Cohort} = USPopulationHP^{2030}_{Age\ Cohort}$$

- Calculating the projected state population for 2030 from exponential growth.

$$StatePopulationExp_{2030} = StatePopulation_{2020} * e^{r*10}$$

- Raking the state population using the national total

$$\boldsymbol{StatePopulation_{2030}} = \frac{StatePopulationExp_{2030}}{\sum_{States} StatePopulationExp_{2030}} * USPopulation_{2030}$$

This is the final state level population projection for 2030.

- 2040 projections ($\boldsymbol{USPopulation_{2040}}$ ***and*** $\boldsymbol{StatePopulation_{2040}}$) are generated using the same approach and repeating the previous process.

ADJUSTMENT

Comparing the growth of this decade (between 2010-2015) versus the average of the growth over the last 100 years (decennial growth from 1910-2010), we identify North Dakota's current growth rate is not plausible to continue.

We still use the current rate for projecting North Dakota's population in 2020. However, for 2030 and 2040, we apply the rate over the longer (and more stable) time horizon between the population count of 1 April 2000 and population estimate of 1 July 2015.

$$StatePopulation_{2015} = StatePopulation_{2000} * e^{Adj\_r * 15.25}$$

which implies the rate is

$$Adj\_r = \frac{1}{15.25} * \ln(\frac{StatePopulation_{2015}}{StatePopulation_{2000}})$$

The adjusted population projection for the specific state would be,

$$Adj\ddot{S}tatePopulation_{2030} = S\ddot{t}atePopulation_{2020} * e^{Adj\_r * 10}$$

$$Adj\ddot{S}tatePopulation_{2040} = S\ddot{t}atePopulation_{2030} * e^{Adj\_r * 10}$$

The adjusted population projection for the U.S. would be,

$$AdjUSPopulation_{2030} = USPopulation_{2030} - S\ddot{t}atePopulation_{2030} + Adj\ddot{S}tatePopulation_{2030}$$

$$AdjUSPopulation_{2040} = USPopulation_{2040} - S\ddot{t}atePopulation_{2040} + Adj\ddot{S}tatePopulation_{2040}$$

POPULATION PROJECTIONS BY AGE

For projecting the population by age, we apply the state-specific CPRs and CCRs to age forward the 2010 population for each state. These ratios are developed identically to the methodology for the US described earlier in this document. This generates the proper Hamilton Perry age distribution for 2020 within each state, which can then be applied to the state population for 2020 calculated earlier,

in order to get the projected population by age for 2020 for the 50 states and D.C. For the U.S. as a whole, population in each age cohort can be calculated by summing the corresponding population belonging to the age group across each of the states.

This process is repeated for 2030 and 2040 by applying the Hamilton Perry age forwarding to the state population in the immediately preceding decade, and using this age distribution to redistribute the previously calculated state total projections. The national projected population by age for 2030 and 2040 is similarly calculated by summing over the projected age categories across all the states.

POPULATION PROJECTIONS BY GENDER

Projections by gender are determined by maintaining the population's age-specific sex-ratio as per the 2010 census. This sex-ratio is applied to the projected population in each age cohort within each state, for 2020, 2030 and 2040. So the gender projections are controlled to the subcategories within the state population projections. The male and female population groups within each age cohort are then summed across all the states, to get the U.S. population projections by gender.

NOTES:

1. We do not use Hamilton Perry age-forwarding for projecting individual state populations because the cohort change ratios of individual states may fluctuate rapidly, even over short time intervals.

2. We do not apply exponential growth rates directly to the U.S. total population because the rate is based off of a short interval of 5.25 years, which may be unstable.

3. The U.S. total population for 2020 is the sum total of the state total populations for 2020 which are projected using exponential growth rates for the individual states.

4. $\{\sum_{States} StatePopulationExp_{2020} = \sum_{States} StatePopulation_{2020} = USPopulation_{2020}\}$

$$\neq \{\sum_{Age\ Cohorts} USPopulationHP^{2020}_{Age\ Cohort} = USPopulationHP_{2020}\}$$

5. $\{\sum_{Age\ Cohorts} USPopulationHP^{2030}_{Age\ Cohort} = USPopulationHP_{2030} = USPopulation_{2030}$

$$= \sum_{States} StatePopulation_{2030}\ \} \neq \{\sum_{States} StatePopulationExp_{2030}\}$$

6. For the U.S. total population for 2030 and 2040, we age forward the 2020 population projections as per Hamilton Perry method. The national totals are then distributed among the 50 states and D.C. according to the state shares in the state level projections developed using exponential growth rates. This ensures that sum of the state level population projections are controlled to the national total population projections developed with the Hamilton-Perry method.

REFERENCES:

Hamilton, C. Horace, and Josef Perry. 1962. "A Short Method for Projecting Population by Age from One Decennial Census to Another." *Social Forces*, 41: 163-70.